2.2.18 Longitudinal and transverse spherical aberrations

A light beam parallel to the optic axis reaches an inner circular (radius r_1) surface of a planoconvex lens (R = 20 mm, t = 3 mm, n = 1.67). A maximum value for r_1 must be defined sufficient to satisfy the condition of paraxial rays within an error of 2%. Then define the interval (z'_{min}, z'_{max}) of the longitudinal aberration and, for the transverse aberration, the area of the disk (radius r_2) of the real image (Fig. 2.55). Why a geometrical point can't be a real image?

Solution:

Since for α (in radiant) less than 20° there is $\alpha = \sin\alpha = \tan\alpha$ with an error less than 2% it can be assumed α_{max} = 20°. Let use y_{max} for r_1 given by (Fig. 2.56)

$$y_{max} = r_1 = R\sin\alpha_{max} = 6.8\,\text{mm}$$

Dividing the interval (0, y_{max}) into N=21 values of α (= 0°, 1°, . . ., 20°) (Fig. 2.57), the corresponding values of y, d and α' are

$$y = R\sin\alpha \qquad d = R(1-\cos\alpha) \qquad \alpha' = \arcsin\left(\frac{\sin\alpha}{n}\right)$$

On the plane surface of the lens the rays are incident with an angle β and emerge with α''. Hence

$$\beta = \alpha - \alpha' \qquad n\sin\beta = \sin\alpha'' \qquad \alpha'' = \arcsin(n\sin\beta)$$

and (Fig. 2.58)

$$y - y' = (t-d)\tan\beta \qquad y' = y - (t-d)\tan\beta$$

Rays emerging from the lens intersect the optic axis at different point distant z' from the plane surface of the lens

$$y' = z'\tan\alpha'' \qquad z' = y'/\tan\alpha''$$

To two of N values y_i and y_{i+1} correspond the quadruplet E'_i , α''_i , z'_i, y''_i, and E'_{i+1} , α''_{i+1} , z'_{i+1}, y''_{i+1} (Fig. 2.59).

The first term of z'_i is 28.1 mm and the last (N = 21) term is z'_N = 26.9 mm.

The focus of the lens is

$$P = \frac{1}{f} = \frac{n-1}{R} \qquad f = 29.9\,\text{mm}$$

The value z'_1 is surely the best one satisfying the condition of paraxial rays. In fact it corresponds to α = 0°. Then for the principal point H we can write

$$h = -(f - z_1) = -1.8\,\text{mm}$$

If we assume s = 10 mm as the distance from position z'_N to the place of a screen where the point image is observed, we have

$$y''_N = \frac{y'_N}{z'_N}s = 2.5\,\text{mm}$$

and for the area D of the image disk of radius r_2

$$r_2 = y''_N = 2.5\,\text{mm} \qquad D = \pi r_2^2 = 19\,\text{mm}^2$$

The final image must have an area different from zero because there is finite the value of the intensity of light.

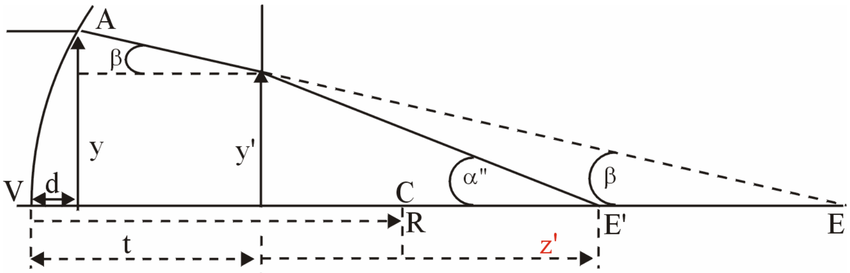

Fig. 2.58

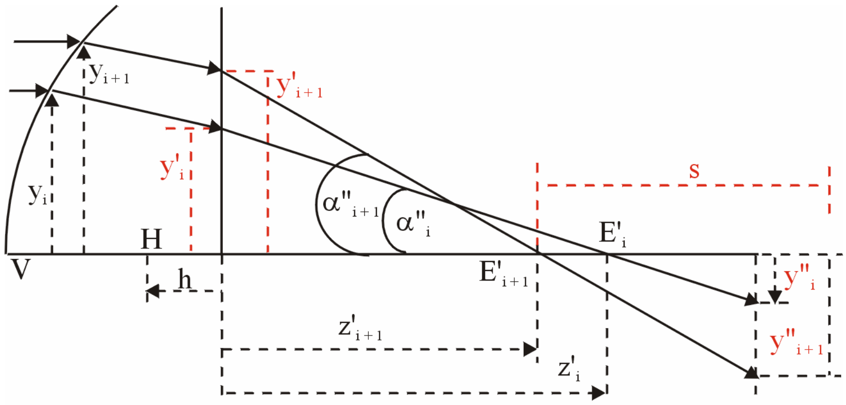

Fig. 2.59